5. A. R. Zak and M. L. Williams, "Crack point stress singularities at bimaterial interface," J. Appl. Mech., 30, No. 1 (1963).
6. G. S. Pisarenko, V. P. Naumenko, and G. S. Volkov, Determination of the Fracture Toughness of Materials on the Basis of the Energy Contour Integral [in Russian], Naukova Dumka, Kiev (1978).
7. A. N. Zlatin and A. A. Khrapkov, "Semi-infinite crack parallel to the boundary of an elastic half-space," Dokl. Akad. Nauk SSSR, 291, No. 4 (1986).

APPLICATION OF METHODS OF THE MECHANICS OF HETEROGENEOUS MEDIA TO DESCRIBE DISPERSION PROCESSES IN AN ELECTROMAGNETIC FIELD

S. Yu. Arutyunov, I. N. Dorokhov,
V. V. Kafarov, V. G. Korniichuk,
and V. P. Solov'ev

UDC 538.4:621.926.085.54-185

The authors of [1, 2] found equations describing the thermohydromechanics of a two-phase polydisperse medium which take into account refinement of the particles of the disperse phase. Here, we use the methods of the mechanics of heterogeneous media [3] and fundamental equations of electrodynamics [4] to obtain a mathematical description of the refinement process in an electromagnetic field (EMF) which considers the collision, destruction, and formation of disperse-phase particles and the effect of the EMF on these events.

1. The assumptions used in [1-3] are adopted here to study the motion of a heterogeneous mixture of three phases in an EMF. The first phase is the carrier phase (liquid or gas), while the second and third phases are in the form of individual particles of a material undergoing refinement and bodies of different sizes undergoing fragmentation.

We introduce the volume contents of the phase α_i and the mean densities ρ_i ($i = 1, 2, 3$) at each point of the volume occupied by the mixture:

$$\alpha_1 + \int_0^R r f_2(r)\,dr + \int_{R_1}^{R_2} \mu f_3(\mu)\,d\mu = 1, \quad \rho = \rho_1 + \int_0^R \rho_2^0 r f_2(r)\,dr + \int_{R_1}^{R_2} \rho_3^0 \mu f_3(\mu)\,d\mu.$$

Here, the polydispersity of the second phase is characterized by the function $f_2(r)dr$ [the number of particles undergoing refinement per unit volume whose dimensions (volumes) are within the range $(r, r + dr)$], while the polydispersity of the third phase is characterized by the function $f_3(\mu)d\mu$ [the number of bodies undergoing fragmentation per unit volume whose dimensions (volumes) are within the interval $(\mu, \mu + d\mu)$]; the subscripts 1, 2, and 3 pertain to the carrier phase, the disperse phase, and the phase comprised of bodies being fragmented; $R_1 > R$. Following [1, 2], we introduce the notion of an r-phase as an aggregate of particles whose dimensions lie within the interval $(r, r + dr)$. We also introduce the notion of a μ-phase as an aggregate of fragmenting particles whose dimensions lie within the interval $(\mu, \mu + d\mu)$. Each phase represents a charged, electrically-conducting, polarized, and magnetized medium in the EMF.

2. In constructing models of continua interacting with an EMF, various methods of formulating the equations of electrodynamics can be used – depending on the expressions used for the local field strengths $\mathbf{E}_1, \mathbf{E}_2(r), \mathbf{E}_3(\mu)$ and $\mathbf{H}_1, \mathbf{H}_2(r), \mathbf{H}_3(\mu)$. However, after one of these formulations is chosen, all of the conservation laws of mechanics must be considered with allowance for this formulation. The formulation of the equations of electrodynamics currently in widest use is the Chu model [5]. We assume that

$$\mathbf{E}_1^* + \mu_0 \mathbf{H}_1 \times \mathbf{v}_1 = \mathbf{E}_1, \quad \mathbf{H}_1^* - \varepsilon_0 \mathbf{E}_1 \times \mathbf{v}_1 = \mathbf{H}_1,$$
$$\mathbf{E}_2^*(r) + \mu_0 \mathbf{H}_2(r) \times \mathbf{v}_2(r) = \mathbf{E}_2(r), \quad \mathbf{H}_2^*(r) - \varepsilon_0 \mathbf{E}_2(r) \times \mathbf{v}_2(r) = \mathbf{H}_2(r),$$
$$\mathbf{E}_3^*(\mu) + \mu_0 \mathbf{H}_3(\mu) \times \mathbf{v}_3(\mu) = \mathbf{E}_3(\mu), \quad \mathbf{H}_3^*(\mu) - \varepsilon_0 \mathbf{E}_3(\mu) \times \mathbf{v}_3(\mu) = \mathbf{H}_3(\mu).$$

Then the electrodynamical equations take the form

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 140-145, July-August, 1988. Original article submitted May 6, 1987.

0021-8944/88/2904-0583$12.50

$$\mathrm{rot}\,\mathbf{E}_1 + \frac{\partial\mu_0\mathbf{H}_1}{\partial t} = \frac{\partial\mu_0\mathbf{M}_1}{\partial t} - \mathrm{rot}\,(\mu_0\mathbf{M}_1\mathbf{v}_1) - \boldsymbol{\psi}_1 - \lambda_1\mathbf{v}_1,$$

$$\mathrm{rot}\,\mathbf{H}_1 - \frac{\partial\varepsilon_0\mathbf{E}_1}{\partial t} = \frac{\partial\mathbf{P}_1}{\partial t} + \mathrm{rot}\,(\mathbf{P}_1 \times \mathbf{v}_1) + \mathbf{i}_1 + \boldsymbol{\gamma}_1 + \eta_1\mathbf{v}_1,$$

$$\mathrm{div}(\mu_0\mathbf{H}_1) = \lambda_1 - \mathrm{div}(\mu_0\mathbf{M}_1),\ \mathrm{div}(\varepsilon_0\mathbf{E}_1) = q_1 + \eta_1 + \mathrm{div}\,\mathbf{P}_1,$$

$$\mathrm{rot}\,\mathbf{E}_2(r) + \frac{\partial\mu_0\mathbf{H}_2(r)}{\partial t} = \frac{\partial\mu_0\mathbf{M}_2(r)}{\partial t} - \mathrm{rot}\,[\mu_0\mathbf{M}_2(r)\times\mathbf{v}_2(r)] - \boldsymbol{\psi}_2(r) - \lambda_2(r)\,\mathbf{v}_2(r),$$

$$\mathrm{rot}\,\mathbf{H}_2(r) - \frac{\partial\varepsilon_0\mathbf{E}_2(r)}{\partial t} = \frac{\partial\mathbf{P}_2(r)}{\partial t} + \mathrm{rot}\,[\mathbf{P}_2(r\times\mathbf{v}_2\ (r)] + \mathbf{i}_2(r) + \boldsymbol{\gamma}_2(r) + \eta_2(r)\,\mathbf{v}_2(r),$$

$$\mathrm{div}(\mu_0\mathbf{H}_2(r)) = \lambda_2(r) - \mathrm{div}(\mu_0\mathbf{M}_2(r)),$$

$$\mathrm{div}(\varepsilon_0\mathbf{E}_2(r)) = q_2(r) + \eta_2(r) + \mathrm{div}\mathbf{P}_2(r),$$

$$\mathrm{rot}\,\mathbf{E}_3(\mu) + \frac{\partial\mu_0\mathbf{H}_3(\mu)}{\partial t} = \frac{\partial\mu_0\mathbf{M}_3(\mu)}{\partial t} - \mathrm{rot}\,[\mu_0\mathbf{M}_3(\mu)\times\mathbf{v}_3(\mu)] - \boldsymbol{\psi}_3(\mu) - \lambda_3(\mu)\,\mathbf{v}_3(\mu),$$

$$\mathrm{rot}\,\mathbf{H}_3(\mu) - \frac{\partial\varepsilon_0\mathbf{E}_3(\mu)}{\partial t} = \frac{\partial\mathbf{P}_3(\mu)}{\partial t} + \mathrm{rot}\,[\mathbf{P}_3(\mu)\times\mathbf{v}_3(\mu)] + \mathbf{i}_3(\mu) + \boldsymbol{\gamma}_3(\mu) + \eta_3(\mu)\,\mathbf{v}_3(\mu),$$

$$\mathrm{div}(\mu_0\mathbf{H}_3(\mu)) = \lambda_3(\mu) - \mathrm{div}(\mu_0\mathbf{M}_3(\mu)),$$

$$\mathrm{div}(\varepsilon_0\mathbf{E}_3(\mu)) = q_3(\mu) + \eta_3(\mu) - \mathrm{div}\,\mathbf{P}_3(\mu),$$

where $\mathbf{E}_1^*$, $\mathbf{E}_2^*(r)$, $\mathbf{E}_3^*(\mu)$, $\mathbf{H}_1^*$, $\mathbf{H}_2^*(r)$, $\mathbf{H}_3^*(\mu)$ are effective values of the local strengths of the electric and magnetic fields in the carrier, r-, and μ-phases measured in a system of reference moving at the velocities $\mathbf{v}_1$, $\mathbf{v}_2(r)$, $\mathbf{v}_3(\mu)$; ε_0 and μ_0 are the permittivity and permeability; $\boldsymbol{\psi}_1, \boldsymbol{\psi}_2(r), \boldsymbol{\psi}_3(\mu)$, λ_1, $\lambda_2(r)$, $\lambda_3(\mu)$ are the magnetic current and charges in each phase, respectively; $\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2(r), \boldsymbol{\gamma}_3(\mu)$, η_1, $\eta_2(r)$, $\eta_3(\mu)$ are the additional electric currents and charges in each phase; $\mathbf{i}_1$, $\mathbf{i}_2(r)$, $\mathbf{i}_3(\mu)$, q_1, $q_2(r)$, $q_3(\mu)$ are the electric currents and charges in the phases; $\mathbf{P}_1$, $\mathbf{P}_2(r)$, $\mathbf{P}_3(\mu)$, $\mathbf{M}_1$, $\mathbf{M}_2(r)$, $\mathbf{M}_3(\mu)$ are polarization and magnetization vectors in the phases.

3. We will introduce the probability $A(r, \mu, \xi)$ of destruction of a particle of the r-phase by two fragmenting bodies ξ and the μ-phase in their collision. We also introduce the probability density $B(r, \gamma, \mu, \xi)$ of the formation of a particle of the r-phase in the destruction of a γ-phase particle by two fragmenting bodies ξ and the μ-phase in their collision.

Then the equations of conservation of the mass of the carrier, r-, and μ-phases take the form

$$\frac{\partial\rho_1}{\partial t} + \mathrm{div}\,(\rho_1\mathbf{v}_1) = 0; \tag{3.1}$$

$$\frac{\partial}{\partial t}\left[\rho_3^0\mu f_3(\mu)\,d\mu\right] + \mathrm{div}\left[\rho_3^0\mu f_3(\mu)\,\mathbf{v}_3(\mu)\,d\mu\right] = 0; \tag{3.2}$$

$$\frac{\partial}{\partial t}\left[\rho_2^0 r f_2(r)\,dr\right] + \mathrm{div}\left[\rho_2^0 r f_2(r)\,\mathbf{v}_2(r)\,dr\right] =$$

$$= -\left[\rho_2^0 r f_2(r)\,dr\right]\int\limits_{R_1}^{R_2}\int\limits_{\mu}^{R_2} A(r, \mu, \xi)\,d\xi\,d\mu + \int\limits_{r}^{R}\int\limits_{R_1}^{R_2}\int\limits_{\mu}^{R_2} A(\gamma, \mu, \xi) B(r, \gamma, \mu, \xi) f_2(\gamma)\,d\xi\,d\gamma\,dr. \tag{3.3}$$

The first term on the right side of Eq. (3.3) characterizes the reduction in the mass of the r-phase due to destruction of particles with the volume r. The second term characterizes the increase in the mass of the r-phase due to the formation of particles with the volume r in the destruction of particles with the volume γ. The momentum conservation equations for the phases in differential form appear similar to the form in [1].

4. We will write the momentum conservation equations of the phases in the form

$$\rho_1\frac{dJ_1\boldsymbol{\omega}_1}{dt} = \mathrm{div}\,G_1 + I\times\tau_1 - \int\limits_0^R \rho_2^0 r f_2(r)\,\mathbf{L}_{12}dr - \int\limits_{R_1}^{R_2}\rho_3^0\mu f_3(\mu)\,\mathbf{L}_{13}d\mu +$$

$$+ \int\limits_{R_1}^{R_2}\int\limits_{\mu}^{R_2}\rho_3^0\mu K(\mu, \xi)\,f_3(\mu)\,f_3(\xi)\,J_3(\mu)\left[\boldsymbol{\omega}_3(\mu) - \boldsymbol{\omega}_3^*(\mu, \xi)\right]d\xi\,d\mu + \int\limits_0^R\int\limits_{R_1}^{R_2}\int\limits_{\mu}^{R_2}\rho_2^0 r f_2(r)\,A(r, \mu, \xi)\,J_2(r)\,\boldsymbol{\omega}_2(r)\,d\xi\,d\mu\,dr -$$

$$- \int\limits_0^R \rho_2^0 r \int\limits_r^R\int\limits_{R_1}^{R_2}\int\limits_{\mu}^{R_2} A(\gamma, \mu, \xi)\,B(r, \gamma, \mu, \xi)\,f_2(\gamma)\,J_2(r)\,\boldsymbol{\omega}_2'(r, \gamma, \mu, \xi)\,\times$$

$$\times\, d\mu\, d\gamma\, d\xi\, dr + \rho_1 \mathbf{p}_1 \times \mathbf{E}_1^* + \mu_0 \rho_1 \mathbf{m}_1 \times \mathbf{H}_1^*,$$

$$f_2(r)\frac{d\boldsymbol{\omega}_2(r)J_2(r)}{dt} = f_2(r)\mathbf{L}_{12} - \frac{1}{\rho_2^0 r}\int_{R_1}^{R_2}\mathbf{L}_{23}(r,\mu)\,d\mu +$$

$$+\int_r^R\int_{R_1}^{R_2}\int_\mu^{R_2} A(\gamma,\mu,\xi)B(r,\gamma,\mu,\xi)f_2(\gamma)J_2(r)\left[\boldsymbol{\omega}_2'(r,\gamma,\mu,\xi) - \boldsymbol{\omega}_2(r)\right]d\xi\,d\mu\,d\gamma +$$

$$+ f_2(r)\mathbf{p}_2(r)\times\mathbf{E}_2^*(r) + \mu_0 f_2(r)\mathbf{m}_2(r)\times\mathbf{H}_2^*(r),$$

$$f_3(\mu)\frac{d\boldsymbol{\omega}_3(\mu)J_3(\mu)}{dt} = f_3(\mu)\mathbf{L}_{13} + \frac{1}{\rho_3^0\mu}\int_0^R\mathbf{L}_{23}(r,\mu)\,dr +$$

$$+\int_{R_1}^{R_2}K(\mu,\xi)f_3(\mu)f_3(\xi)J_3(\mu)\left[\boldsymbol{\omega}_3^*(\mu,\xi)-\boldsymbol{\omega}_3(\mu)\right]d\xi + f_3(\mu)\mathbf{p}_3(\mu)\times\mathbf{E}_3^*(\mu) + \mu_0 f_3(\mu)\mathbf{m}_3(\mu)\times\mathbf{H}_3^*(\mu).$$

Here, J_1, $J_2(r)$, $J_3(\mu)$ are the moments of inertia per unit mass of the carrier, r-, and μ-phases, respectively; $\boldsymbol{\omega}_1$, $\boldsymbol{\omega}_2(r)$, $\boldsymbol{\omega}_3(\mu)$ are vectors of the angular velocity of the carrier, r-, and μ-phases; $\boldsymbol{\omega}_3^*(\mu,\xi)$ is the vector of the angular velocity of a fragmenting body of the volume μ after an inelastic collision with another fragmenting body of the volume ξ; $\boldsymbol{\omega}_2'(r,\gamma,\mu,\xi)$ is the vector of angular velocity of a particle of volume r formed in the destruction of a particle of volume γ; $\mathbf{L}_{12}$, $\mathbf{L}_{13}$, $\mathbf{L}_{23}(r,\mu)$ are the moments of interaction per unit mass between the phases; G_1 is the tensor of the moment stresses in the carrier phase.

5. We will adopt the hypothesis of additivity of the main thermodynamic and electrodynamic characteristics with respect to the masses of the phases. Proceeding similarly to [1-3], we obtain differential equations for the internal energies of the phases:

$$\rho_1\frac{dU_1}{dt} = \int_0^R\rho_2^0 r f_2(r)q_{21}(r)\,dr + \int_{R_1}^{R_2}\rho_3^0\mu f_3(\mu)q_{31}(\mu)\,d\mu - \operatorname{div} q_1 - p\rho_1\frac{d}{dt}\left(\frac{1}{\rho_1^0}\right) +$$

$$+\int_0^R\rho_2^0 r f_2(r)\mathbf{f}_{21}[\mathbf{v}_2(r)-\mathbf{v}_1]\,dr + \int_{R_1}^{R_2}\rho_3^0\mu f_3(\mu)f_{31}[\mathbf{v}_3(\mu)-\mathbf{v}_1]\,d\mu +$$

$$+\int_0^R\rho_2^0 r f_2(r)\mathbf{L}_{12}[\boldsymbol{\omega}_1-\boldsymbol{\omega}_2(r)]\,dr + \int_{R_1}^{R_2}\rho_3^0\mu f_3(\mu)\mathbf{L}_{13}[\boldsymbol{\omega}_1-\boldsymbol{\omega}_3(\mu)]\,d\mu + \tau_1 : \operatorname{Grad}\boldsymbol{\omega}_1 - \boldsymbol{\omega}_1(I\times\tau_1) +$$

$$+\int_0^R\int_{R_1}^{R_2}\int_\mu^{R_2}\rho_2^0 r f_2(r)A(r,\mu,\xi)\left\{\frac{[\mathbf{v}_2(r)-\mathbf{v}_1]^2}{2} + \frac{J_2(r)[\boldsymbol{\omega}_2(r)-\boldsymbol{\omega}_1]^2}{2}\right\}d\xi\,d\mu\,dr +$$

$$+\int_{R_1}^{R_2}\int_\mu^{R_2}\rho_3^0\mu K(\mu,\xi)f_3(\mu)f_3(\xi)\left\{\frac{[\mathbf{v}_3(\mu)-\mathbf{v}_1]^2}{2} + \frac{J_3(\mu)[\boldsymbol{\omega}_3(\mu)-\boldsymbol{\omega}_1]^2}{2}\right\}d\xi\,d\mu +$$

$$+\,\mathbf{i}_1^*\mathbf{E}_1 + \left(\rho_1\frac{d\mathbf{p}_1}{dt} - \boldsymbol{\omega}_1\times\rho_1\mathbf{p}_1\right)\cdot\mathbf{E}_1^* + \left(\rho_1\frac{d\mu_0\mathbf{m}_1}{dt} - \boldsymbol{\omega}_1\times\rho_1\mu_0\mathbf{m}_1\right)\cdot\mathbf{H}_1^*,$$

$$\rho_2^0 r f_2(r)\frac{dU_2(r)}{dt} = -\rho_2^0 r f_2(r)q_{21}(r) - \int_{R_1}^{R_2}q_{23}(r,\mu)\,d\mu +$$

$$+\int_0^R\int_{R_1}^{R_2}\int_\mu^{R_2}A(\gamma,\mu,\xi)B(r,\gamma,\mu,\xi)f_2(\gamma)[U_2(\gamma)-U_2(r)]\,d\xi\,d\mu\,d\gamma -$$

$$-\,p\rho_2^0 r f_2(r)\frac{d}{dt}\left(\frac{1}{\rho_2^0}\right) + \int_{R_1}^{R_2}f_{32}(r,\mu)[\mathbf{v}_3(\mu)-\mathbf{v}_2(r)]\,d\mu$$

$$+\int_{R_1}^{R_2}\mathbf{L}_{23}(r,\mu)[\boldsymbol{\omega}_2(r)-\boldsymbol{\omega}_3(\mu)]\,d\mu +$$

$$+\,\rho_2^0 r\int_r^R\int_{R_1}^{R_2}\int_\mu^{R_2}A(\gamma,\mu,\xi)B(r,\gamma,\mu,\xi)f_2(\gamma)\left\{\frac{\left[\mathbf{v}_2'(r,\gamma,\mu,\xi)-\mathbf{v}_2(r)\right]^2}{2} -\right.$$

$$-\frac{[\mathbf{v}_2(r,\gamma,\mu,\xi)-\mathbf{v}_1]^2}{2}+\frac{J_2(r)\left[\boldsymbol{\omega}_2'(r,\gamma,\mu,\xi)-\boldsymbol{\omega}_2(r)\right]^2}{2}-$$

$$-\frac{J_2(r)\left[\boldsymbol{\omega}_2'(r,\gamma,\mu,\xi)-\boldsymbol{\omega}_1\right]^2}{2}\Bigg\}d\xi\,d\mu\,d\gamma+\int\limits_{R_1}^{R_2}\int\limits_{\mu}^{R_2}\rho_2^0 rf_2(r)A(r,\mu,\xi)\left\{\frac{\varkappa\varepsilon_0}{2\rho_2}\left[\mathbf{E}_1^2+\right.\right.$$

$$\left.+\mathbf{E}_2^2(r)+\mathbf{E}_3^2(\mu)\right]+\frac{\varkappa\mu_0}{2\rho_2}\left[\mathbf{H}_1^2+\mathbf{H}_2^2(r)+\mathbf{H}_3^2(\mu)\right]+\mu_0\mathbf{m}_2(r)\mathbf{H}_2^*(r)+$$

$$+\mathbf{p}_2(r)\mathbf{E}_2^*(r)\Big\}d\xi\,d\mu-\rho_2^0 r\int\limits_r^R\int\limits_{R_1}^{R_2}\int\limits_{\mu}^{R_2}A(r,\mu,\xi)B(r,\gamma,\mu,\xi)f_2(\gamma)\left\{\frac{\varkappa\mu_0}{2\rho_2}\left[\mathbf{H}_1^2+\right.\right.$$

$$\left.+\mathbf{H}_2^2(r)+\mathbf{H}_3^2(\mu)\right]+\frac{\varkappa\varepsilon_0}{2\rho_2}\left[\mathbf{E}_1^2+\mathbf{E}_2^2(r)+\mathbf{E}_3^2(\mu)\right]+\mu_0\mathbf{m}_2(r)\mathbf{H}_2^*(r)+$$

$$+\mathbf{p}_2(r)\mathbf{E}_2^*(r)\Big\}d\xi\,d\mu\,d\gamma+\mathbf{i}_2^*(r)\mathbf{E}_2(r)+\left[\rho_2^0 rf_2(r)\frac{d\mathbf{p}_2(r)}{dt}-\right.$$

$$\rho_3^0\mu f_3(\mu)\frac{dU_3(\mu)}{dt}=-\rho_3^0\mu f_3(\mu)q_{31}(\mu)+\int\limits_0^R q_{23}(r,\mu)\,dr-$$

$$\left.-\boldsymbol{\omega}_2(r)\times\rho_2^0 rf_2(r)\mathbf{p}_2(r)\right]\cdot\mathbf{E}_2^*(r)+\left[\rho_2^0 rf_2(r)\frac{d\mu_0\mathbf{m}_2(r)}{dt}-\boldsymbol{\omega}_2(r)\times\rho_2^0 rf_2(r)\mu_0\mathbf{m}_2(r)\right]\cdot\mathbf{H}_2^*(r),$$

$$\rho_3^0\mu f_3(\mu)\frac{dU_3(\mu)}{dt}=-\rho_3^0\mu f_3(\mu)q_{31}(\mu)+\int\limits_0^R q_{23}(r,\mu)\,dr-$$

$$-p\rho_3^0\mu f_3(\mu)\frac{d}{dt}\left(\frac{1}{\rho_3^0}\right)+\int\limits_0^R\int\limits_{\mu}^{R_2}\rho_2^0 rf_2(r)A(r,\mu,\xi)U_2(r)\,d\xi\,dr-\int\limits_0^R\rho_2^0 r\int\limits_r^R\int\limits_{\mu}^{R_2}A(\gamma,\mu,\xi)B(r,\gamma,\mu,\xi)f_2(\gamma)U_2(\gamma)\,d\xi\,d\gamma\,dr+$$

$$+\int\limits_{R_1}^{R_2}\rho_3^0\mu K(\mu,\xi)f_3(\mu)f_3(\xi)\left\{\frac{[\mathbf{v}_3^*(\mu,\xi)-\mathbf{v}_1]^2}{2}-\frac{[\mathbf{v}_3^*(\mu,\xi)-\mathbf{v}_3(\mu)]^2}{2}+\right.$$

$$\left.+\frac{J_3(\mu)\left[\boldsymbol{\omega}_3^*(\mu,\xi)-\boldsymbol{\omega}_1\right]^2}{2}-\frac{J_3(\mu)\left[\boldsymbol{\omega}_3^*(\mu,\xi)-\boldsymbol{\omega}_3(\mu)\right]^2}{2}\right\}d\xi+\mathbf{i}_3^*(\mu)\mathbf{E}_3^*(\mu)+$$

$$+\left[\rho_3^0\mu f_3(\mu)\frac{d\mathbf{p}_3(\mu)}{dt}-\boldsymbol{\omega}_3(\mu)\times\rho_3^0\mu f_3(\mu)\mathbf{p}_3(\mu)\right]\cdot\mathbf{E}_3^*(\mu)+\left[\rho_3^0\mu f_3(\mu)\frac{d\mu_0\mathbf{m}_3(\mu)}{dt}-\boldsymbol{\omega}_3(\mu)\times\rho_3^0\mu f_3(\mu)\mu_0\mathbf{m}_3(\mu)\right]\mathbf{H}_3^*(\mu),$$

where the sign (:) denotes a scalar product of two tensors; q_1 is the external heat flux per unit volume of the mixture; $q_{21}(r)$, $q_{31}(\mu)$, $q_{23}(r,\mu)$ are the specific heat fluxes between the phases; $U_2(r)$, $U_2(\gamma)$ is the internal energy of disperse-phase particles of the volumes r and γ.

Thus, we have obtained a mathematical description of a system (carrier phase, particles undergoing refinement, fragmenting bodies, EMF) which describes the motion of a three-phase, charged, polarized, and magnetized mixture in an EMF with allowance for refinement.

6. Additivity of the entropy of the mixture follows from the additivity hyopothesis stated earlier. Then, using the Gibbs relations, we write the expression for the entropy derivative of the mixture

$$\rho\frac{DS}{Dt}=\rho\frac{D_iS}{Dt}+\sum_{i=1}^{2}\sum_{j=i+1}^{3}\sigma_1(i,j)+\sigma_2+\sum_{i=1}^{2}\sum_{j=i+1}^{3}\sigma_3(i,j)+$$

$$+\sum_{i=1}^{2}\sum_{j=i+1}^{3}\sigma_4(i,j)+\sigma_5+\sigma_6+\sigma_7+\sum_{i=1}^{3}\sigma_8(i)+\sum_{i=1}^{3}\sigma_9(i)+\sigma_{10}+\sigma_d+\sigma_f. \tag{6.1}$$

The last two terms in (6.1) characterize the entropy of the mixture resulting directly from the destruction and formation of particles. The driving forces in the destruction of particles of the r-phase and in the formation of r-phase particles with the destruction of γ-phase particles have the form

$$X_p=\rho_2^0 r\left[\frac{F_2(r)}{T_2(r)}-\frac{F_1}{T_1}\right]+\rho_2^0 rU_2(r)\left[\frac{1}{T_3(\mu)}-\frac{1}{T_2(r)}\right]+\rho_2^0 r\left\{\frac{[\mathbf{v}_2(r)-\mathbf{v}_1]^2}{2}+\frac{J_2(r)[\boldsymbol{\omega}_2(r)-\boldsymbol{\omega}_1]^2}{2}\right\}+$$

$$\left\{\frac{\rho_2 r\varkappa\varepsilon_0}{2T_2(r)\rho_2}\left[\mathbf{E}_1^2+\mathbf{E}_2^2(r)+\mathbf{E}_3^2(\mu)\right]+\frac{\rho_2^0 r\varkappa\mu_0}{2\rho_2T_2(r)}\left[\mathbf{H}_1^2+\mathbf{H}_2^2(r)+\mathbf{H}_3^2(\mu)\right]+\frac{\rho_2^0 r}{T_2(r)}\left[\mu_0\mathbf{m}_2(r)\mathbf{H}_2^*(r)+\mathbf{p}_2(r)\mathbf{E}_2^*(r)\right]\right\}; \tag{6.2}$$

$$
\begin{aligned}
X_0 = \rho_2^0 r \Bigg\{ \left[\frac{F_2(\gamma)}{T_2(\gamma)} - \frac{F_1}{T_1} \right] + \left[\frac{F_2(r)}{T_2(r)} - \frac{F_2(\gamma)}{T_2(\gamma)} \right] + \rho_2^0 r U_2(\gamma) \left[\frac{1}{T_3(\mu)} - \frac{1}{T_2(r)} \right] + \rho_2^0 r \Bigg\{ \frac{[\mathbf{v}_2'(r, \gamma, \mu, \xi) - \mathbf{v}_1]^2}{2T_2(r)} - \\
- \frac{[\mathbf{v}_2'(r, \gamma, \mu, \xi) - \mathbf{v}_2(r)]^2}{2T_2(r)} + \frac{J_2(r)[\boldsymbol{\omega}_2'(r, \gamma, \mu, \xi) - \boldsymbol{\omega}_1]^2}{2T_2(r)} - \\
- \frac{J_2(r)[\boldsymbol{\omega}_2'(r, \gamma, \mu, \xi) - \boldsymbol{\omega}_2(r)]^2}{2T_2(r)} \Bigg\} + \Bigg\{ \frac{\rho_2^0 r \varkappa \varepsilon_0}{2\rho_2 T_2(r)} [\mathbf{E}_1^2 + \mathbf{E}_2^2(r) + \mathbf{E}_3^2(\mu)] + \\
+ \frac{\rho_2 \varkappa \mu_0}{2\rho_2 T_2(r)} [\mathbf{H}_1^2 + \mathbf{H}_2^2(r) + \mathbf{H}_3^2(\mu)] + \frac{\rho_2^0 r}{T_2(r)} [\mu_0 \mathbf{m}_2(r) \mathbf{H}_2^*(r) + \mathbf{p}_2(r) \mathbf{E}_2^*(r)] \Bigg\}. \qquad (6.3)
\end{aligned}
$$

The experimental studies [6, 7] demonstrated the dependence of the refinement process in an EMF on the strength of the disperse-phase particles, the energy of the fragmenting bodies, and the energy of the electromagnetic field. This finding is consistent with the structure of the equations (6.2), (6.3) obtained here for the driving forces.

LITERATURE CITED

1. V. V. Kafarov, I. N. Dorokhov, et al., "Motion of a polydisperse two-phase mixture with allowance for fragmentation of the inclusions," Teor. Osn. Khim. Tekhnol., 17, No. 3 (1983).
2. V. V. Kafarov, I. N. Dorokhov, and S. Yu. Arutyunov, "Systematic analysis of processes in chemical engineering," in: Refinement and Mixing of Bulk Materials [in Russian], Nauka, Moscow (1985).
3. R. I. Nigmatulin, Principles of the Mechanics of Heterogeneous Media [in Russian], Nauka, Moscow (1978).
4. K. T. Téi, "On the electrodynamics of moving media," TIIÉR, 52, No. 3 (1964).
5. R. M. Fano, L. T. Chu, and R. B. Adler, Electromagnetic Fields, Energy, and Forces. Selection A 1.5.1, Wiley, N. Y. (1960).
6. V. A. Vlasov and N. B. Kotlyarova, "Optimum conditions for the refinement of materials in an electromagnetic apparatus," Steklo Keram., No. 3 (1984).
7. V. A. Abrosimov, Yu. N. Kuznetsov, et al., "New method of dispersing pigment suspensions," Lakokras. Mater. Ikh Primen., No. 4 (1980).

TEMPERATURE DEPENDENCE OF THE DYNAMIC STIFFNESS OF MATERIALS

B. I. Abashkin, I. Kh. Zabirov,
E. F. Krapivina, and V. I. Semenov

UDC 620.172:620.171.3

Several investigations have examined the effect of temperature on the cleavage strength of structural materials. In connection with certain technical problems encountered in realizing intensive shock loading and performing the necessary measurements, in conditions of high temperature, the authors of [1] did not consider the effect of the temperature of preheating on the form of the equation of state, while it was argued in [2] that the reduction in pressures and tensile stresses due to changes in the properties of materials at high temperatures is negligible.

Here, we derive a formula to evaluate dynamic stiffness for a material from its known coefficient of thermal expansion and we propose an empirical method of obtaining this formula for any material by studying the cleavage fracture which takes place in the collision of cold and hot specimens in a gas gun. We also compare calculations and experimental results for aluminum alloy AMg6.

In experimentally studying the temperature dependence of the cleavage strength of materials by the method of high-speed collision of a striker with a hot target made of the test material, it is necessary to determine the following slightly increasing function of temperature

$$
f(T) = \frac{Z_0}{Z_T} = \frac{(1 + 3\alpha T) C_0}{(1 + 3\alpha T_0) C_T} \simeq (1 + 3\alpha \Delta T) \frac{C_0}{C_T} \simeq \frac{C_0}{C_T}, \qquad (1)
$$

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 4, pp. 145-148, July-August, 1988. Original article submitted March 25, 1987.